Exercise 32

Find $f'(a)$.

$$f(t) = 2t^3 + t$$

Solution

Determine the derivative of $f(t)$.

$$f'(t) = \lim_{h \to 0} \frac{f(t + h) - f(t)}{h}$$

$$= \lim_{h \to 0} \frac{[2(t + h)^3 + (t + h)] - [2t^3 + t]}{h}$$

$$= \lim_{h \to 0} \frac{[2(t^3 + 3t^2h + 3th^2 + h^3) + t + h] - 2t^3 - t}{h}$$

$$= \lim_{h \to 0} \frac{(2t^3 + 6t^2h + 6th^2 + 2h^3 + t + h) - 2t^3 - t}{h}$$

$$= \lim_{h \to 0} \frac{6t^2h + 6th^2 + 2h^3 + h}{h}$$

$$= \lim_{h \to 0} (6t^2 + 6th + 2h^2 + 1)$$

$$= 6t^2 + 1$$

Plug in $t = a$ to this formula to get $f'(a)$.

$$f'(a) = 6a^2 + 1$$